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Naturalness of nonlinear σ, ω self-couplings in relativistic mean-field models for neutron stars

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Abstract. We investigate dense hadronic matter in a relativistic mean-field approach. For a generalized baryon-meson Lagrangian effective field theory, we confront expansions from naive dimensional analysis for the nonlinear self-couplings of the σ, ω fields with estimates from microscopic $q\bar{q}$ pair creation models from Quantum Chromodynamics. Upon adjusting the model parameters to ordinary nuclear matter, we discuss implications of the approach to dense hadronic matter and in particular to neutron stars.

PACS. 11.10.-z Field theory – 21.65.+f Nuclear matter – 26.60.+c Nuclear matter aspects of neutron stars

The investigation of hadronic matter is presently one of the leading topics in nuclear physics. One efficient approach to dense hadronic matter is based on Quantum Hadrodynamics (QHD) [1]: within the framework of effective meson and baryon degrees of freedom, the nuclear many-body problem is treated in a relativistic mean-field approach. This has turned out as a very economical parameterization: in the simplest approximation, keeping only the Hartree self-energy, classical σ and ω fields exhaust the overwhelming part of the effective NN interaction in the nuclear medium at ordinary nuclear matter density $\rho_0 \cong 0.15\,\mathrm{fm}^{-3}$.

Exploring hadronic matter at higher densities, the original Walecka Lagrangian [1] has to be extended significantly: besides the excitation of hyperons or the inclusion of leptons e^-, μ^- for charge neutrality, in particular nonlinear self couplings of the σ- and ω-mesons become increasingly important, as genuine many-body forces are expected to dominate with increasing matter density. A natural way to classify their contribution is to expand the Lagrangian density in terms of the characteristic scales of QCD. Here different expansion schemes are possible: fundamental scales are the renormalization invariant parameter $\Lambda_{\mathrm{QCD}} \sim 200\,\mathrm{MeV}$ or an expansion in the number of colours of quarks, N_c, reminiscent of the $SU(3)$ group structure of QCD. However, focusing on meson and baryons as effective low-energy degrees of freedom (equivalently realized in the large-N_c limit) as a result of chiral-symmetry breaking, the appropriate scales are the low-energy chiral parameters of QCD, *i.e.* the weak pion decay constant f_π and the chiral parameter Λ_χ: $f_\pi = 93\,\mathrm{MeV}$, $\Lambda_\chi \sim 1\,\mathrm{GeV}$, $\Lambda_\chi \le 4\pi f_\pi$. The corresponding interaction Lagrangian is then given as [2]

$$L_{\mathrm{eff}} = \left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M}\right)^\ell \sum_{i,k=0}^{\infty} \frac{c_{i,k}}{i!k!}\left(\frac{\sigma}{f_\pi}\right)^i \left(\frac{\omega}{f_\pi}\right)^k f_\pi^2 \Lambda_\chi^2\,, \qquad (1)$$

with unknown expansion coefficients $c_{i,k}$. While there is evidence for ordinary nuclear matter that the expansion in the nonlinear mesonic couplings quickly converges —keeping only the cubic and quartic couplings of the σ-meson provides a semi-quantitative fit to nuclear matter data [3]— a controlled and useful extension to significantly higher densities ($\rho \ge 5\rho_0$) requires some assumption on a natural ordering of the expansion coefficients. Evidently, at least two schemes allow a compact summation of the full expansion series, *i.e.*

$$c_{i,k} = 1 \longrightarrow L_{\mathrm{eff}} = \left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M}\right)^\ell \exp\left(\frac{\sigma}{f_\pi}\,\frac{\omega}{f_\pi}\right), \qquad (2)$$

or, alternatively,

$$c_{i,k} = i!k! \longrightarrow L_{\mathrm{eff}} = \left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M}\right)^\ell \frac{1}{1+(\sigma/f_\pi)}\,\frac{1}{1+(\omega/f_\pi)}\,. \qquad (3)$$

A more phenomenological, but otherwise more flexible parameterization for nuclear matter, which combines the two limits derived above, taking into account σ-, ω- and

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ϱ-meson fields, is given as [4,5]

$$\begin{aligned}\mathcal{L} = &\sum_B \bar{\psi}_B \left[i\gamma_\mu \partial^\mu - (M_B - g^\star_{\sigma B}\sigma) - g^\star_{\omega B}\gamma_\mu \omega^\mu \right]\psi_B \\ &- \sum_B \psi_B \left[\frac{1}{2} g^\star_{\varrho B}\gamma_\mu \boldsymbol{\tau}\cdot\boldsymbol{\varrho}^\mu \right]\psi_B + \sum_\lambda \bar{\psi}_\lambda [i\gamma_\mu\partial^\mu - m_\lambda]\psi_\lambda \\ &+ \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma{}^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu \\ &- \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{2}m_\varrho^2 \boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu ,\end{aligned}$$

where $g^\star_{\sigma B} \equiv m^\star_{\alpha B} g_\sigma$; $g^\star_{\omega B} \equiv m^\star_{\beta B} g_\omega$; $g^\star_{\varrho B} = m^\star_{\gamma B} g_\varrho$ and $m^\star_{nB} \equiv (1 + \frac{g_\sigma \sigma}{n M_B})^{-n}$; $n = \alpha, \beta, \gamma$. We assume, guided by phenomenology, α, β and γ as real and positive numbers (similar interaction terms may be associated to the vector and isovector sector of the Lagrangian density). Notice that we have assumed as a convenient starting point a universal coupling by setting $g_{(\sigma,\omega,\varrho)B} \to g_{(\sigma,\omega,\varrho)}$; the exponential and geometrical series above are recovered in the limit $n \to \infty$ and $n \to 1$, respectively. Evidently the parameterization above is identified with the ansatz in ref. [4] for the sigma-meson, upon rescaling the expansion parameters as

$$\frac{\sigma}{f_\pi} \cong \frac{g_\sigma \sigma}{\Lambda_\chi} \cong \frac{g_\sigma\sigma}{M}, \tag{4}$$

where M denotes the bare mass of the nucleon.

The crucial question for the relevance of the parameterizations formulated above is the naturalness of the coefficients: for example, for a rigorous summation in the exponential form, the coefficients have to be strictly equal to 1. In practice, such a constraint is never met; however, dimensional analysis and re-summation of infinite series might be still qualitatively an organizational scheme to control the expansion of —otherwise completely arbitrary— effective Lagrangians and to define their continuation to higher densities. With a full QCD calculation presently beyond any reach, one approach is to evolve the couplings of higher orders from a perturbative loop expansion based on effective intermediate mesonic degrees of freedom; alternative are (still rather phenomenological) constituent quark and effective non-perturbative $q\bar{q}$ pair creation models, which take into account the quark structure of the interacting mesons.

Focusing in this note briefly on the second route, among the various QCD-inspired models for $q\bar{q}$ production, we are currently pursuing the investigation of 3 different parameterizations: the effective one-gluon exchange ${}^3S_1$ model [6], the instanton-induced $q\bar{q}$ excitation [7] and the vacuum pair creation ${}^1P_0$ model [8]. In their structural form these models are very similar with their characteristic $q\bar{q}$ operator

$$L_{q\to q(q\bar{q})} \approx c(a,b) \lim_{\gamma\to\infty} \frac{\mathrm{d}}{\mathrm{d}\gamma}\delta(\boldsymbol{r}_i - \boldsymbol{r}_k + \boldsymbol{\sigma}_k\gamma); \tag{5}$$

where the main difference is contained in the operator $c_{a,b}$ involving the colours of the interacting quarks. In practice, the ansatz above is significantly simplified in the zero-range approximation for the radial form factor which is reduced to a δ-function in coordinate space For typical mass scales of $m_g \sim 0.3\,\mathrm{fm}^{-1}$ for non-perturbative gluon exchange or instanton models, the zero-range limit seems qualitatively well justified; finite-range corrections are readily estimated. For the mesonic structure a simple $q\bar{q}$ quark configuration is assumed:

$$\phi^{1-}_\omega(\boldsymbol{r}) = N_\omega e^{-\alpha_\omega r^2}\left[\frac{1}{2}\frac{\bar{1}}{2}\right]^{1M}_S \left[\frac{1}{2}\frac{\bar{1}}{2}\right]^{\infty}_F [(10)(01)]^\infty_C , \tag{6}$$

$$\phi^{0^+}_\sigma = N_\sigma r e^{-\alpha_\sigma r^2}\left[Y_1(\hat{r})\left[\frac{1}{2}\frac{1}{2}\right]^1\right]^\infty_S \left[\frac{1}{2}\frac{1}{2}\right]^\infty_F [(10)(01)]^\infty_C , \tag{7}$$

for the omega and sigma as ${}^3S_1$ and ${}^3P_0$ states, respectively, with a typical size parameter $\alpha_{\sigma,\omega} = \frac{1}{2b^2}$; $b \simeq 0.6\,\mathrm{fm} \sim \sqrt{\frac{3}{8}\langle r^2_\lambda\rangle}$. Above the brackets in angular momentum, flavor and color space denote the standard coupling of the quarks to the (external) quantum numbers of the corresponding meson. The corresponding effective nonlinear coupling constants are then easily extracted in comparison with the corresponding overlap matrix elements, schematically, $g_{\sigma\lambda\lambda} = \langle[\phi_\lambda\phi_\lambda]^\infty|\sum L_{q\to q(q\bar{q})}|\phi_\sigma\rangle$ with $\lambda = \sigma,\omega$, where the sum includes pair creation from the quark and anti-quark of the σ-meson in the initial state.

Actually, only preliminary results can be reported: with current values of the strong quark-gluon or the instanton coupling constant, which enter as typical parameters into the calculation, we find in cubic order qualitatively a natural behaviour of the non-linearities within a factor 2; this trend is qualitatively confirmed to higher orders by just investigating the relative re-coupling strengths of the nonlinear vertices. However, in particular for the σ-meson, these findings have to take with care: in view of the highly unknown structure of the σ as the lightest scalar mesons with its (compared to the glue ball-scale) very low mass and its extremely strong coupling to the $\pi\pi$ channel (which suggests a dominant $(q\bar{q})(q\bar{q})$ 4-quark component), further detailed investigations are indispensable. Deferring details, we just mention an economical alternative to the problem above: the quark-meson coupling model QMC [9]. In this model the interaction of mesons as quasi-elementary objects is reduced by construction to a simple counting scheme. Different investigations of the baryon-meson and meson-meson interactions predict a natural ordering of the nonlinear couplings [10]. As a consistent generalization of the QMC to the meson-meson interactions is still an open problem, further investigations have to be awaited for.

We explore the consequences of natural expansions for dense hadronic matter. Of course, here still the problem of the continuation of the vertices to higher densities arises. In our present approach, we neglect as an exploratory step the density dependence both in the mean-field potential and the effective baryon masses

$$\begin{aligned} U_{\mathrm{Hartree}} &\cong -\left(\frac{g_\sigma^2}{m_\sigma^2} - \frac{g_\omega^2}{m_\omega^2}\right)\rho(r); \\ M^* &= M - g_\sigma\sigma \cong M - \frac{g_\sigma^2}{m_\sigma^2}\rho(r). \end{aligned} \tag{8}$$

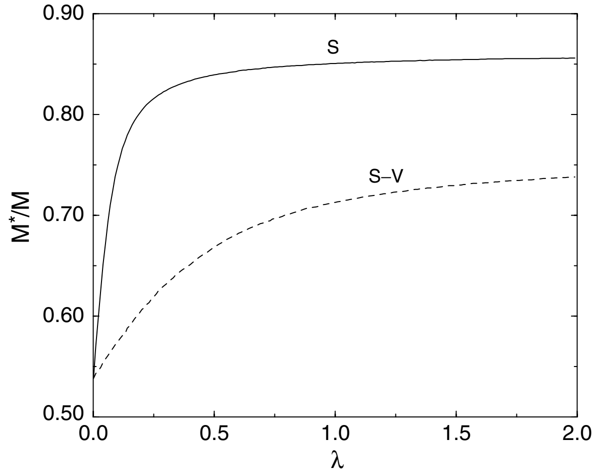

Fig. 1. Ratio M^*/M as a function of λ for cases S (variations of λ and $\alpha = \beta = 0$) (full line) and S-V ($\lambda = \alpha = \beta = 0$) (dashed line).

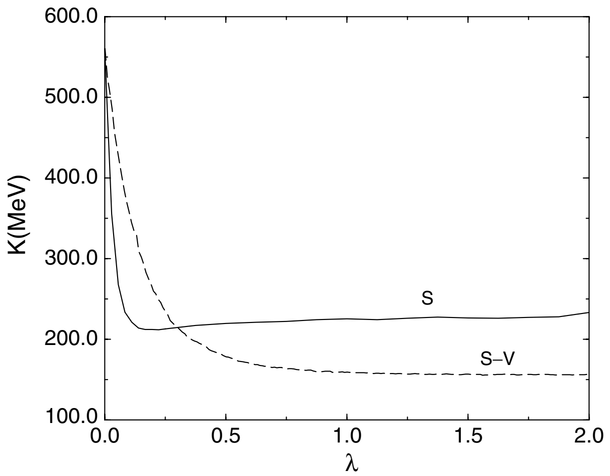

Fig. 2. Compression modulus of nuclear matter K as a function of λ for cases S (variations of λ and $\alpha = \beta = 0$) (full line) and S-V ($\lambda = \alpha = \beta = 0$) (dashed line).

Then applying the model to standard nuclear matter up to neutron stars we solve a system of transcendental equations taking into account chemical equilibrium, baryon number and electrical charge conservation, which determines the equation of state (EOS) for the system considered. The Lagrangian density of our approach includes the eight octet baryons coupled to three mesons (σ, ω, ϱ), and two free lepton species ($\ell = e^-, \mu^-$), with he scalar, vector and iso-vector coupling constants determined to reproduce bulk nuclear matter properties (see for example [4]). Typical results of our approach on the summation of the nonlinear couplings in eq. (5), are presented in figs. 1-3 for the effective nucleon mass in the medium, the compressibility and the maximum mass of a neutron star, showing a substantial sensitivity. In particular, our finding of maximum neutron star masses up to $1.8M_\odot$ in the S case seems to support recent observations of masses well above the typical masses of $1.4M_\odot$ (see references in [4,5]).

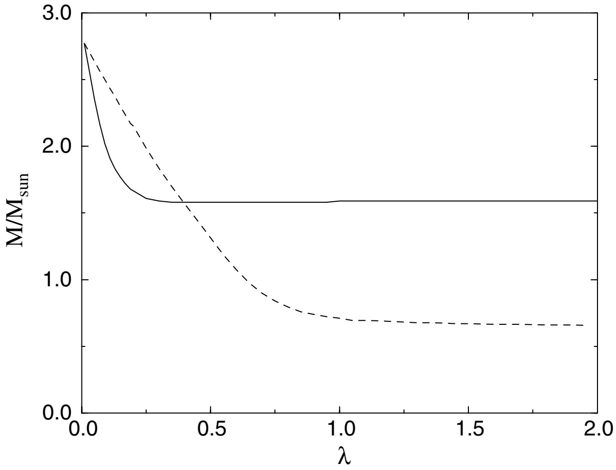

Fig. 3. Maximum neutron star mass as a function of the λ parameter (full line: S case; dashed line: S-V case).

In summarizing our brief comment, we feel that the implementation of dimensional analysis in effective field theories is a promising strategy. Further detailed and extensive model calculations are however indispensable to establish a natural continuation of nonlinear expansions to hadronic densities as typical for new forms of nuclear matter.

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